Automatic Straight Line Detection through Fixed-Point BYY Harmony Learning

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Abstract. Straight line detection in a binary image is a basic but difficult task in image processing and machine vision. Recently, a fast fixed-point BYY harmony learning algorithm has been established to efficiently make model selection automatically during the parameter learning on Gaussian mixture. In this paper, we apply the fixed-point BYY harmony learning algorithm to learning the Gaussians in the dataset of a binary image and utilize the major principal components of the covariance matrices of the estimated Gaussians to represent the straight lines in the image. It is demonstrated well by the experiments that this fixed-point BYY harmony learning approach can both determine the number of straight lines and locate these straight lines accurately in a binary image.

Keywords: Straight line detection, Bayesian Ying-Yang (BYY) harmony learning, Gaussian mixture, Automated model selection, Major principal component.

1 Introduction

Detecting straight lines from a binary image is a basic task in image processing and machine vision. In the pattern recognition literature, a variety of algorithms have been proposed to solve this problem. The Hough transform (HT) and its variations (see Refs. [1,2] for reviews) might be the most classical ones. However, this kind of algorithms usually suffer from large time and space requirements, and detection of false positives, even if the Random Hough Transform (RHT) [3] and the constrained Hough Transform [4] have been proposed to overcome these weaknesses. Later on, there appeared many other algorithms for straight line or curve detection (e.g., [5,6]), but most of these algorithms need to know the number of straight lines or curves in the image in advance.

With the development of the Bayesian Ying-Yang (BYY) harmony learning system and theory [7,8,9,10], a new kind of learning algorithms [11,12,13,14,15] have been established for the Gaussian mixture modeling with a favorite feature that model selection can be made automatically during parameter learning. From the view of line detection, a straight line can be recognized as the major

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principal component of the covariance matrix of certain flat Gaussian of black pixels since the number of black pixels along it is always limited in the image. In such a way, these BYY harmony learning algorithms can learn the Gaussians from the image data automatically and detect the straight lines with the major principal components of their covariance matrices. On the other hand, from the BYY harmony learning on the mixture of experts in [16], a gradient learning algorithm was already proposed for the straight line or ellipse detection, but it was applicable only for some simple cases.

In this paper, we apply the fixed-point BYY harmony learning algorithm [15] to learning an appropriate number of Gaussians and utilize the major principal components of the covariance matrices of these Gaussians to represent the straight lines in the image. It is demonstrated well by the experiments that this fixed-point BYY harmony learning approach can efficiently determine the number of straight lines and locate these straight lines accurately in a binary image.

In the sequel, we introduce the fixed-point BYY harmony learning algorithm and present our new straight line detection approach in Section 2. In Section 3, several experiments on both the simulation and real images are conducted to demonstrate the efficiency of our BYY harmony learning approach. Finally, we conclude briefly in Section 4.

2 Fixed-Point BYY Harmony Learning Approach for Automatic Straight Line Detection

2.1 Fixed-Point BYY Harmony Learning Algorithm

As a powerful statistical model, Gaussian mixture has been widely applied in the fields of information processing and data analysis. Mathematically, the probability density function (pdf) of the Gaussian mixture model of k components in $\Re^d$ is given as follows:

$$\Phi(x) = \sum_{i=1}^{k} \alpha_i q(x|\theta_i), \qquad \forall x \in \Re^d, \tag{1}$$

where $q(x|\theta_i)$ is a Gaussian pdf with the parameters $\theta_i = (m_i, \Sigma_i)$, being given by

$$q(x|\theta_i) = q(x|m_i, \Sigma_i) = \frac{1}{(2\pi)^{\frac{n}{2}}|\Sigma_i|^{\frac{1}{2}}} e^{-\frac{1}{2}(x-m_i)^T \Sigma_i^{-1}(x-m_i)}, \tag{2}$$

and $\alpha_i(\geq 0)$ are the mixing proportions under the constraint $\sum_{i=1}^{k} \alpha_i = 1$. If we encapsulate all the parameters into one vector: $\Theta_k = (\alpha_1, \alpha_2, \ldots, \alpha_k, \theta_1, \theta_2, \ldots, \theta_k)$, then, according to Eq.(1), the pdf of the Gaussian mixture can be rewritten as:

$$\Phi(x|\Theta_k) = \sum_{i=1}^{k} \alpha_i q(x|\theta_i) = \sum_{i=1}^{k} \pi_i q(x|m_i, \Sigma_i). \tag{3}$$

For the Gaussian mixture modeling, there have been several statistical learning algorithms, including the EM algorithm [17] and the k-means algorithm [18]. However, these approaches require an assumption that the number of Gaussians in the mixture is known in advance. Unfortunately, this assumption is practically unrealistic for many unsupervised learning tasks such as clustering or competitive learning. In such a situation, the selection of an appropriate number of Gaussians must be made jointly with the estimation of the parameters, which becomes a rather difficult task [19].

In fact, this model selection problem has been investigated by many researchers from different aspects. The traditional approach was to choose a best number k^* of Gaussians in the mixture via certain model selection criterion, such as Akaike's information criterion (AIC) [20] and the Bayesian Information Criterion (BIC) [21]. However, all the existing theoretic selection criteria have their limitations and often lead to a wrong result. Moreover, the process of evaluating a information criterion or validity index incurs a large computational cost since we need to repeat the entire parameter estimation process at a large number of different values of k. In the middle of 1990s, there appeared some stochastic approaches to infer the mixture model. The two typical approaches are the methods of reversible jump Markov chain Monte Carlo (RJMCMC) [22] and the Dirichlet processes [23], respectively. But these stochastic simulation methods generally require a large number of samples with different sampling methods, not just a set of sample data. Actually, it can efficiently solved through the BYY harmony learning on a BI-architecture of the BYY learning system related to the Gaussian mixture. Given a sample data set $\mathcal{S} = \{x_t\}_{t=1}^N$ from a mixture of k^* Gaussians, the BYY harmony learning for the Gaussian mixture modeling can be implemented by maximizing the following harmony function:

$$J(\Theta_k) = \frac{1}{N} \sum_{t=1}^{N} \sum_{j=1}^{k} \frac{\alpha_j q(x_t \mid \theta_j)}{\sum_{i=1}^{k} \alpha_i q(x_t \mid \theta_j)} ln[\alpha_j q(x_t \mid \theta_j)] \tag{4}$$

where $q(x \mid \theta_j)$ is a Gaussian mixture density given by Eq.(2).

For implementing the maximization of the harmony function, some gradient learning algorithms as well as an annealing learning algorithm were already established in [11,12,13,14]. More recently, a fast fixed-point learning algorithm was proposed in [15]. It was demonstrated well by the simulation experiments on these BYY harmony learning algorithms that as long as k is set to be larger than the true number of Gaussians in the sample data, the number of Gaussians can be automatically selected for the sample data set, with the mixing proportions of the extra Gaussians attenuating to zero. That is, these algorithms owns a favorite feature of automatic model selection during the parameter learning, which was already analyzed and proved for certain cases in [24]. For automatic straight line detection, we will apply the fixed-point BYY harmony learning algorithm to maximizing the harmony function via the following iterative procedure:

$$\alpha_j^+ = \frac{\sum_{t=1}^N h_j(t)}{\sum_{i=1}^i \sum_{t=1}^N h_i(t)}; \tag{5}$$

$$m_j^+ = \frac{1}{\sum_{t=1}^N h_j(t)} \sum_{t=1}^N h_j(t) x_t; \tag{6}$$

$$\Sigma_j^+ = \frac{1}{\sum_{t=1}^N h_j(t)} \sum_{t=1}^N h_j(t)(x_t - \hat{m}_j)(x_t - \hat{m}_j)^T, \tag{7}$$

where $h_j(t) = p(j|x_t) + \sum_{i=1}^k p(j|x_t)(\delta_{ij} - p(j|x_t)) ln[\alpha_i q(x_t|m_i, \Sigma_i)]$, $p(j|x_t) = \alpha_j q(x_t|m_j, \Sigma_j) / \sum_{i=1}^k \alpha_i q(x_t|m_i, \Sigma_i)$ and δ_{ij} is the Kronecker function. It can be seen from Eqs (5)-(7) that the fixed-point BYY harmony learning algorithm is very similar to the EM algorithm for Gaussian mixture. However, since $h_j(t)$ introduces a rewarding and penalizing mechanism on the mixing proportions [13], it differs from the EM algorithm and owns the favorite feature of automated model selection on Gaussian mixture.

2.2 The Proposed Approach to Automatic Straight Line Detection

Given a set of black points or pixels $\mathcal{B} = \{x_t\}_{t=1}^N$ ($x_t = [x_{1,t}, x_{2,t}]^T$) in a binary image, we regard the black points along each line as one flat Gaussian distribution. That is, those black points can be assumed to be subject to a 2-dimensional Gaussian mixture distribution. Then, we can utilize the fixed-point BYY harmony learning algorithm to estimate those flat Gaussians and use the major principal components of their covariance matrices to represent the straight lines as long as k is set to be larger than the number k^* of the straight lines in the image. In order to speed up the convergence of the algorithm, we set a threshold value $\delta > 0$ such that as soon as the mixing proportion is lower than δ, the corresponding Gaussian will be discarded from the mixture.

With the convergence of the fixed-point BYY harmony learning algorithm on $\mathcal{B}$ with $k \geq k^*$, we get k^* flat Gaussians with the parameters $\{(\alpha_i, m_i, \Sigma_i)\}_{i=1}^{k^*}$ from the resulted mixture. Then, from each Gaussian $(\alpha_i, m_i, \Sigma_i)$, we pick up m_i and the major principle component $V_{1,i}$ of Σ_i to construct a straight line equation $l_i : U_{1,i}^T(x - m_i) = 0$, where $U_{1,i}$ is the unit vector being orthogonal to $V_{1,i}$, with the mixing proportion α_i representing the proportion of the number of points along this straight line l_i. Since the sample points are in a 2-dimensional space, $U_{1,i}$ can be uniquely determined and easily solved from $V_{1,i}$, without considering its direction. Hence, the problem of detecting multiple straight lines in a binary image has been turned into the Gaussian mixing modeling problem of both model selection and parameter learning, which can be efficiently solved by the fixed-point BYY harmony learning algorithm automatically.

With the above preparations, as $k(> k^*)$, the stop criterion threshold value $\varepsilon(> 0)$ and the component annihilation threshold value $\delta(> 0)$ are all prefixed, the procedure of our fixed-point BYY harmony learning approach to automatic straight line detection with $\mathcal{B}$ can be summarized as follows.

1. Let $t = 1$ and set the initial parameters Θ_0 of the Gaussian mixture as randomly as possible.
2. At time t, update the parameters of the Gaussian mixture at time $t-1$ by Eqs (5)-(7) to get the new parameters $\Theta_t = (\alpha_i, m_i, \Sigma_i)_{i=1}^k$;
3. If $|J(\Theta_t) - J(\Theta_{t-1})| \leq \varepsilon$, stop and get the result Θ_t, and go to Step 5; otherwise, let $t = t+1$ and go to Step 4.
4. If some $\alpha_i \leq \delta$, discard the component $\theta_i = (\alpha_i, m_i, \Sigma_i)$ from the mixture and modify the mixing proportions with the constraint $\sum_{j=1}^k \alpha_j = 1$. Return to Step 2.
5. Pick up m_i and the major principle component $V_{1,i}$ of Σ_i of each Gaussian $(\alpha_j, m_j, \Sigma_j)$ in the resulted mixture to construct a straight line equation $l_i : U_{1,i}^T(x - m_i) = 0$.

It can be easily found from the above automatic straight line detection procedure that the operation of the fixed-point BYY harmony learning algorithm tries to increase the total harmony function on the Gaussian mixture so that the extra Gaussians or corresponding straight lines will be discarded automatically during the parameter learning or estimation.

3 Experimental Results

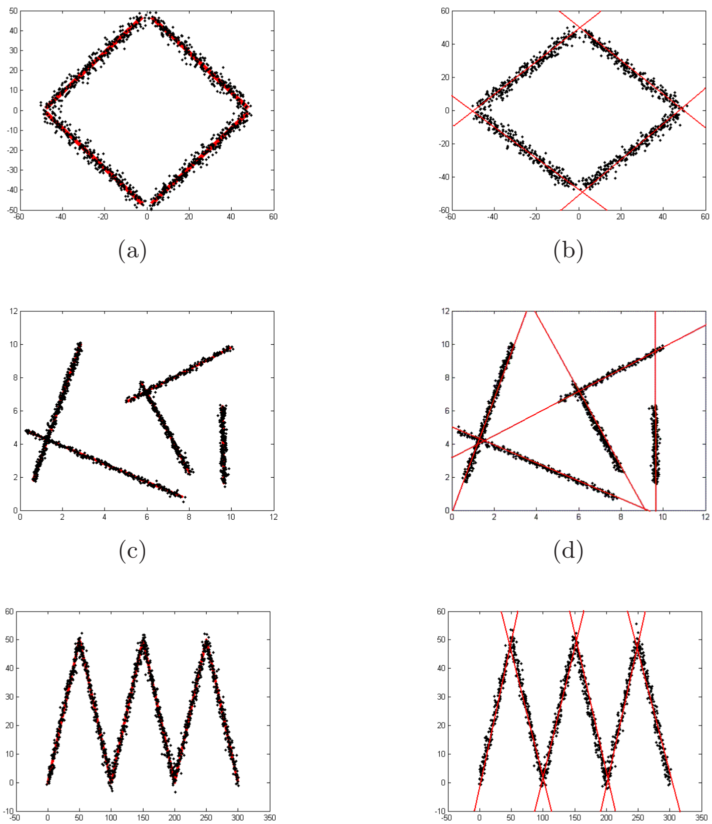
In this section, several simulation and practical experiments are conducted to demonstrate the efficiency of our proposed fixed-point BYY harmony learning approach. In all the experiments, the initial means of the Gaussians in the mixture are trained by the k-means algorithm on the sample data set $\mathcal{B}$. Moreover, the stop criterion threshold value ε is set to be $10 * e^{-8}$ and the component annihilation threshold value δ is set to be 0.08. For clarity, the original and detected straight lines will be drawn with red color, but the sample points along different straight lines will be drawn in black.

3.1 Simulation Results

For testing the proposed approach, simulation experiments are conducted on three binary image datasets consisting of different numbers of straight lines, which are shown in Fig.1(a),(b),(c), respectively. We implement the fixed-point BYY harmony learning algorithm on each of these datasets with $k = 8$. The results of the automatic straight line detection on the three image datasets are shown in Fig.1(d),(e),(f), respectively. Actually, in each case, some random noise from a Gaussian distribution with zero mean and a standard variance 0.2 is added to the coordinates of each black point. It can be seen from the experimental results that the correct number of straight lines is determined automatically to match the actual straight lines accurately in each image dataset.

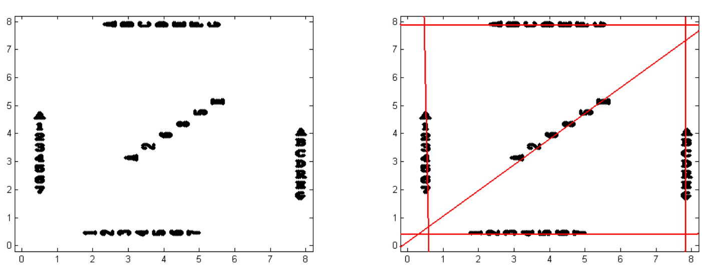
3.2 Automatic Container Recognition

Automatic container recognition system is very useful for customs or logistic management. In fact, our proposed fixed-point BYY harmony learning approach



(a) (b) (c) (d) (e) (f)

Fig. 1. The experiments results of the automatic straight line detection through the fixed-point BYY harmony learning approach. (a),(b),(c) are the three binary image datasets, while (d),(e),(f) are their results of the straight line detection.



(a) (b)

Fig. 2. The experiments results on automatic container recognition. (a) The original container image with five series of numbers (with letters). (b) The result of the automatic container recognition through the fixed-point BYY harmony learning approach.

can be applied to assisting to establish such a system. Container recognition is usually based on the captured container number located at the back of the container. Specifically, the container, as shown in Fig.2(a), can be recognized by the five series of numbers (with letters). The recognition process consists of two steps. The first step is to locate and extract each rectangular area in the raw image that contains a series of numbers, while the second step is to actually recognize these numbers via some image processing and pattern recognition techniques.

For the first step, we implement the fixed-point BYY learning algorithm to roughly locate the container numbers via detecting the five straight lines through the five series of the numbers, respectively. As shown in Fig.2(b), these five straight lines can locate the series of numbers very well. Based on the detected strip lines, we can extract the rectangular areas of the numbers from the raw image. Finally, the numbers can be subsequently recognized via some image processing and pattern recognition techniques.

4 Conclusions

We have investigated the straight line detection in a binary image from the point of view of the Bayesian Ying-Yang (BYY) harmony learning and proposed the fixed-point BYY harmony learning approach to automatic straight line detection. Actually, we implement the fixed-point BYY harmony learning algorithm to learn a number of flat Gaussians from an image dataset automatically to represent the black points along the actual straight lines, respectively, and locate these straight lines with the major principal components of the covariance matrices of the obtained Gaussians. It is demonstrated well by the experiments on the simulated and real-world images that this fixed-point BYY harmony learning approach can both determine the number of straight lines and locate these straight lines accurately in an image.

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