Hörmander's hypoelliptic theorem for nonlocal operators

Xicheng Zhang

Wuhan University

Abstract: In this paper we show the full Hörmander's hypoelliptic theorem for nonlocal operators by a purely probabilistic method: the Malliavin calculus. Roughly speaking, under general Hörmander's Lie bracket condition, we show the regularization effect of discontinuous Lévy noises for possibly degenerate stochastic differential equations with jumps. To treat the large jumps, we use a perturbation argument by interpolation techniques and some short time asymptotic estimates of the semigroup. As an application, we show the existence of fundamental solutions for operator $\partial_t - \mathscr{L}$, where $\mathscr{L}$ is the nonlocal kinetic operators:

$$\mathscr{L} f(x,\mathrm{v}) = \mathrm{p.v}\int_{\mathbb{R}^d}(f(x,\mathrm{v}+\mathrm{v}')-f(x,\mathrm{v}))\frac{\kappa(x,\mathrm{v},\mathrm{v}')}{|\mathrm{v}'|^{d+\alpha}}\mathrm{d}\mathrm{v}' + \mathrm{v}\cdot\nabla_x f(x,\mathrm{v}) + b(x,\mathrm{v})\cdot\nabla_{\mathrm{v}} f(x,\mathrm{v}).$$

Here $\kappa_0^{-1} \leqslant \kappa(x,\mathrm{v},\mathrm{v}') \leqslant \kappa_0$ belongs to $C_b^\infty(\mathbb{R}^{3d})$ and is symmetric in v', p.v. stands for the Cauchy principle value, and $b : \mathbb{R}^{2d} \to \mathbb{R}^d \in C_b^\infty(\mathbb{R}^{2d})$. (This is a joint work with Zimo Hao and Xuhui Peng).